Enrollment No: ______________________ **Exam Seat No: ______________________**

C.U.SHAH UNIVERSITY

Winter Examination-2015

Subject Name: Engineering Mathematics-II

Subject Code: 4TE02EMT2 **Branch: B.Tech(All)**

Semester: II **Date: 19/11/2015**

Time: 10.30 To 1:30 **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 Attempt the following questions: **(14)**

a) $\int_{-\pi/2}^{\pi/2} \sin^7 x \; dx =$ _______

(a) 0 (b) 1 (c) $\frac{\pi}{2}$ (d) $\frac{1}{2}$

b) $\int_{0}^{\pi/2} \cos^4 x \; dx =$ _______

(a) 0 (b) 1 (c) $\frac{3\pi}{16}$ (d) $\frac{8\pi}{3}$

c) $\int_0^1 \int_0^x dy \; dx =$ _______

(a) $\frac{1}{2}$ (b) -1 (c) 0 (d) y

d) The value of $\int_{-\pi}^{\pi} \sin mx \sin nx \; dx$ for $m \neq \pm n$ is

(a) 0 (b) π (c) $\frac{\pi}{2}$ (d) 2π

e) $\beta(1,1) =$ _______

(a) 0 (b) 1 (c) $\sqrt{\pi}$ (d) π

f) $\Gamma(n)\Gamma\left(n+\frac{1}{2}\right) =$ _______

(a) $\frac{\sqrt{\pi}\;\Gamma(2n)}{2^{2n-1}}$ (b) $\frac{\sqrt{\pi}\;\Gamma(2n)}{2^{2n}}$ (c) $\frac{\sqrt{\pi}\;\Gamma(n)}{2^{2n-1}}$ (d) $\frac{\sqrt{\pi}\;\Gamma(n)}{2^{2n}}$

g) If the two tangents at the point are real and coincident, the double point is called __________.

(a) a node (b) a cusp (c) a conjugate point (d) none of these

h) The curve passes through the origin, if the equation does not contain ________

(a) terms in x (b) terms in y (c) constant term (d) none of these

i) Length of curve for $y = f(x)$ is defined by

(a) $\int_{x_1}^{x_2} \sqrt{\left[1+\left(\frac{dy}{dx}\right)\right]}\, dx$ (b) $\int_{x_1}^{x_2} \sqrt{\left[1+\left(\frac{dy}{dx}\right)^2\right]}\, dx$

(c) $\int_{x_1}^{x_2} \sqrt{\left[1+\left(\frac{dx}{dy}\right)^2\right]}\, dx$ (d) $\int_{x_1}^{x_2} \sqrt{\left[1+\left(\frac{dx}{dy}\right)\right]}\, dx$

j) $\int_0^1 \int_1^2 \int_0^3 dx\, dy\, dz =$ ________

(a) 1 (b) -3 (c) $\frac{1}{3}$ (d) 3

k) The degree of the differential equation $\frac{d^2y}{dx^2} = \left[1+\left(\frac{dy}{dx}\right)^3\right]^{\frac{2}{3}}$ is

(a) 1 (b) 2 (c) 3 (d) 6

l) The order of the differential equation $\left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}} = \left[y+5\left(\frac{dy}{dx}\right)\right]^{\frac{1}{2}}$ is

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{3}{2}$ (d) 2

m) The *p*-series $\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+....+\frac{1}{n^p}+....$ is convergent for

(a) $p<1$ (b) $p>1$ (c) $p=1$ (d) none of these

n) The series $1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n}+...$ is

(a) convergent (b) divergent (c) oscillatory (d) none of these

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions

a) Find the volume common to the cylinder $x^2+y^2=a^2$ and $x^2+z^2=a^2$. **(05)**

b) Evaluate: $\int_0^{\pi} x\sin^7 x\cos^4 x\, dx$ **(05)**

c) Prove that (i) $erf_c(x)+erf_c(-x)=2$

(ii) $erf(x)+erf_c(x)=1$ **(04)**

Q-3 Attempt all questions

a) Evaluate: $\int_0^1 \left(x\log\frac{1}{x}\right)^n dx$ **(05)**

b) Test for the convergence the series $\sum_{n=1}^{\infty}\frac{\left[(n+1)x\right]^n}{n^{n+1}}$ **(05)**

c) Solve: $\frac{dy}{dx}+y\tan x=\sin 2x,\ \ y(0)=1$ **(04)**

Q-4 Attempt all questions

a) Test for convergence the series $2+\frac{3}{2}x+\frac{4}{3}x^2+\frac{5}{4}x^3+....$ **(05)**

b) Trace the curve $r=a(1+\cos\theta)$. **(05)**

c) Prove that (i) $n\beta(m+1,n)=m\beta(m,n+1)$

(ii) $\beta(m,n)=\beta(m,n+1)+\beta(m+1,n)$ **(04)**

Q-5 Attempt all questions

a) Evaluate: $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}\left(x^2+y^2+z^2\right)dz\,dy\,dx$ **(05)**

b) Solve: $\frac{dy}{dx}=2y\tan x+y^2\tan^2 x$ **(05)**

c) Test for convergence the series $4-1+\frac{1}{4}-\frac{1}{16}+....$ and if it is convergent then also find its sum. **(04)**

Q-6 Attempt all questions

a) Evaluate $\iint_R xy\ dy\,dx$,where R is the positive quadrant of the circle $x^2+y^2=a^2$ **(05)**

b) Derive Reduction formula for $\int_0^{\frac{\pi}{2}}\cos^n x\,dx,\ \ n\ge 2.$ **(05)**

c) Find the orthogonal trajectories of the family of parabola $y=ax^2$. **(04)**

Q-7 Attempt all questions

a) Change the order of integration and evaluate $\int_0^a \int_{a-\sqrt{a^2-y^2}}^{a+\sqrt{a^2-y^2}} dx\,dy$. **(05)**

b) A circuit containing a resistance R, an inductance L in series is acted on by periodic electromotive force $E\sin\omega t$. If $i=0$ when $t=0$, show that the current at any time t is $i(t)=\frac{E}{\sqrt{R^2+L^2\omega^2}}\left\{\sin(\omega t-\phi)+e^{-\frac{Rt}{L}}\sin\phi\right\}$, where $\phi=\tan^{-1}\left(\frac{L\omega}{R}\right)$ **(05)**

c) Solve: $\left(y^2e^{xy^2}+4x^3\right)dx+\left(2xye^{xy^2}-3y^2\right)dy=0$ **(04)**

Q-8 Attempt all questions

a) Evaluate: $\int_0^{\frac{\pi}{2}} \frac{dx}{\sqrt{2ax-x^2}\sqrt{a^2-x^2}}$ **(05)**

b) Trace the curve $y^2(2a-x)=x^3$. **(05)**

c) Find the perimeter of the cardioid $r=a(1+\cos\theta)$ **(04)**